



## High Dimensional Entangled Systems

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### Summary of first two years activities (October 1, 2008-September 30, 2010)

Entanglement and quantum correlations are the fundamental tools for building protocols of Quantum Information Processing and Communication (QIPC). These fields passed in the last years the proof-of-concept stage, asking now for real-world implementations.

In this context, light plays a fundamental role as the natural carrier of information over large distances and between logic elements within a processor. The standard approach of quantum optics that deals with single or two-mode systems is inadequate in view of practical implementations of QIPC, not only because it is unrealistic, but also because it creates a bottleneck in the information capacity of the quantum channels.

The project HIDEAS aims at a breakthrough in the information capacity of quantum communication, by exploiting the intrinsic multivariate and multi-modal character of the radiation field. The long term vision underlying our project is that of a **broadband quantum communication**, where all the physical properties of the photons are utilized to store information.

The general objectives are to study, on the one hand, how to produce in a controlled way quantum entanglement of light in high dimensional and multimodal spaces, and, on the other hand, how to create multimode quantum interfaces between light and matter so that quantum states of light can be stored and processed in long-lived matter degrees of freedom.

From a different perspective, our research aims at contributing to the field of metrology, a domain where multimode aspects have been introduced with great success ("frequency combs"), and quantum noise tailoring demonstrated a powerful tool to increase the sensitivity of high-precision measurements ("quantum metrology").

The path to reach such general objectives proceeds along five research lines (work packages, WPs) that address both the light (WP1-WP4) and the matter (WP5) aspects, spanning the continuous variable (WP1,WP2) and the photon-pair (WP3,WP4) regimes and encompassing temporal (WP1), spatial (WP2,WP3) and spatio-temporal (WP4) degrees of freedom.

An important tool for broadband information technologies is to convey information by means of the temporal variation of a light beam confined in a well defined spatial mode. Our first work package **WP1, High-D temporal entanglement: quantum frequency combs** studies this field from a quantum perspective. The main objectives are the generation of temporally entangled light (*quantum frequency combs*), the study of how these quantum frequency combs can be used to improve the precision of time and frequency metrology, and of the ways of interfacing such a temporally multimode light with broadband quantum memories.

The first step in this direction is to generate squeezed frequency combs, i.e; trains of short pulses of light with reduced quantum fluctuations on a given field quadrature. Squeezed frequency combs were theoretically shown to be generated by a Synchronously Pumped Optical Parametric Oscillators (SPOPO), i.e an Optical Parametric Oscillator whose cavity length is the same as the femtosecond laser cavity pumping it.

A fundamental achievement of the project is the complete development at UPMC of the SPOPO experimental set-up. A novel technique was used to synchronize optical cavities to frequency combs, which will be patented [Pinel2009]. The SPOPO realized at UPMC works perfectly with a very low threshold (16mW) and has been characterized in detail [Patera2010]. Phase sensitive amplification of injected combs has been observed. As a consequence of deamplification, the SPOPO has been shown to generate intensity squeezing below its threshold with typically -1. dB of squeezing currently achieved. A definite signature of the multimode character of the quantum state was obtained, with a technique based on cutting portions of the spectrum and measuring the amount of squeezing on the transmitted beam.

On the theoretical side, UPMC has shown that non-classical frequency combs can be tailored by appropriately shaping the pump pulses [Patera2009], a technique that looks promising to optimize the time/frequency entanglement in frequency combs. USP and UPMC developed a quantum theory of SPOPOs [Averchenko2010], which predicts that squeezing is present in short time slices inside the pulses, and that quantum correlations exist between nearby pulses. USTRAT found a novel regime of giant sub-threshold pulses driven by quantum fluctuations in SPOPOs [Oppo2009], and predicted that quantum entanglement is observable in such a regime. This opens the possibility of observing macroscopic continuous variable entanglement in SPOPOs.

In the direction of measuring ultra-small time delays, an experimental setup is currently under construction at UPMC, and new techniques of pulse shaping are being developed for optimizing the local oscillator in order to reach the highest possible sensitivity. A fundamental theoretical achievement here is the demonstration that in order to optimize the accuracy in the estimation of any parameter (including a time delay) with multimode non-classical light, the best way is to concentrate all the squeezing available in a single well defined light mode, called the detection mode [Pinel2010]. Pulse shaping techniques of non-classical light is therefore of paramount importance for our purpose.

Images are a privileged way to convey a great deal of information in a parallel way. Recording, storing, processing and displaying images require broadband information channels and large size memories. Our second workpackage **WP2**, *High-D spatial entanglement (continuous variables)*, investigates these problems at the quantum level. In this context, our objectives are i) to study the best ways of generating spatially entangled multimode light, which is the highly-entangled quantum resource necessary for broadband quantum information processing of images, ii) to use these multimode quantum resources to improve various functions of information processing in images (e.g, to enhance the sensitivity in image processing, or in the detection of ultra-small 3D displacements and rotations), and iii) to study the properties of spatially multimode light suitable to be interfaced with multimode quantum memories.

The activity of the first two years has seen several experimental achievements in the generation of spatially entangled multimode light, and we are now in conditions to compare different sources:

-UPMC has set up a self-imaging optical parametric oscillator, which is a self-imaging cavity resonating simultaneously for all the transverse modes at the same frequency, containing a second order crystal pumped with a large beam. In this device, several transverse modes can be independently squeezed and amplified. The multimode capabilities of the device have been assessed showing for the first time frequency doubling of images of arbitrary shapes [Chalopin2010a]. Multimode squeezing effects on three independent transverse modes have been measured [Chalopin2010b], in good agreement with the theory in [Lopez2009] when experimental losses and other imperfections are taken into account. Moreover, it was shown that the oscillation threshold can be crossed while keeping multimode non-classical results [Chalopin2010c].

- ARC, in collaboration with UPMC, implemented an experiment to generate and directly detect for the first time entanglement between two co-propagating transverse modes of a beam of light. Entanglement between two modes was measured and the related work is published in Nature photonics [Janousek2009]. During year 2 lossless non-classical mode conversion and mixing via spatial light modulator was demonstrated [Morizur2010a, Morizur2010b], its application to GHZ state being currently assessed. Mode conversion can be done while preserving the non-classical features, which is of paramount importance for future multimode spatial light manipulation.

-COMO demonstrated sub-shot noise spatial correlations in travelling-wave PDC without background subtraction [Brida2009]. This achievement opens the concrete possibility of realizing imaging with a sensitivity beyond the standard quantum limit.

On the theoretical side, USTL performed a study on the ability to generate N-partite entanglement using N symmetrically-tilted plane waves for pumping a parametric medium [Daems2010]. UPMC produced a detailed review about the application of the powerful tools of symplectic invariants to multimode light [Leroyer 2010], while USTL extended the symplectic theory in order to keep into account the non trivial space-time structure of multimode entanglement [Patera2010b]. With the purpose of studying the problem of interfacing spatially

multimode light with quantum memories USP and UPMC demonstrated that a large number of transverse modes can be stored independently [Golubeva2010], while UKBH solved the full three dimensional problem of light storage in an ensemble and investigated the performance of the memory for storing spatial correlations [Grodecka-Grad2010a,b].

Our **WP3**, *High-D spatial entanglement (photon-pairs)* shares with WP2 the interest towards entanglement in the spatial degrees of freedom, but here the focus is on the two-photon entanglement produced by spontaneous parametric down-conversion (PDC), in the angular degrees of freedom of photons, both in the orbital angular momentum (OAM) and in the angular position domains. The idea is to increase the quantum bandwidth of photonic communication channels by feeding the channels with highly spatially entangled photon pairs, therefore allowing communication with a larger alphabet (qudits) compared to the usual binary alphabet (qubits) allowed by the polarization. The main objective is to demonstrate experimentally high-D entanglement of such photon-pairs (aiming at  $D \approx 50$ ), and to establish its usefulness for quantum communication.

The work of the first two years has seen a number of relevant experimental achievements:

-The UL setup is based on the idea of projecting the two-photon state on a superposition of OAM eigenstates, by using an angular phase plate coupled to a single-photon detector. As a crucial step towards high-D spatial entanglement, the first year focused on the fabrication of the optimum phase plates. The UL team compared several technologies, and found that photolithography lead to superior phase plates. Angular projectors based upon these phase plates were used to study high-D two-photon entanglement, and in particular to test the robustness of OAM entanglement towards propagation in free space [Pors2009]. In year 2 the limits to high-dimensional angular entanglement have been explored; these are determined both by the Schmidt number of the PDC set-up and by the azimuthal/radial geometry of the phase-plate projectors. By optimizing the PDC setup, UL is now able to report a measured Schmidt dimensionality on the order 30-40 [DiLorenzo2010]. Moreover, the UL team demonstrated for the first time the transport of spatially entangled qudits with  $D=3$  (i.e. qutrits) in a hollow-core photonic crystal fiber [Loeffler2010]. This represents important progress towards practical applications of OAM entangled photons in quantum communications.

-The USTRAT setup is based on the use of spatial light modulators for the measurement of the spatial modes of the photons. The step performed in year one involved an up-grade of the PDC source with a mode-locked pump laser, and the demonstration of the efficacy of a silicon photon avalanche diode (SPAD) working at 710 nm. The setup was used to demonstrate a non-local violation of a Bell-type inequality for the orbital angular momentum states of light. In terms of the dimensionality of entanglement, full entanglement has been observed up to a modal index  $\ell=23$  [Leach2009]. Within a ghost imaging setup [Jack2009], edge enhancement in imaging was demonstrated as a direct consequence of the quantum correlations in the orbital angular momentum of photons. Also in this case, a violation of a Bell-type inequality for an OAM subspace was shown. In the second year of the project a true highlight achievement was the demonstration, for the first time to our knowledge, of EPR correlations between down-converted photon pairs in terms of angular position and angular momentum. Correlations an order of magnitude stronger than those allowed by the uncertainty principle to non-entangled particles have been measured [Leach2010]. This result establishes that angular position and angular momentum are suitable variables for applications in quantum information processing, notably in protocols for quantum key distribution.

Mastering the coherence and correlation properties of PDC photon pairs in the spatio-temporal domain is of paramount importance in modern QIPC technologies that relies on the quantum interference of photons. Our fourth research line, **WP4**, *High-D spatio-temporal entanglement: quantum X-waves* addresses a quite peculiar issue, that is, the non factorability in space and time of the spatio-temporal structure of PDC bi-photon entanglement. The idea comes from nonlinear optics, where wave-packets characterized by a X-shaped spatio-temporale profile (X-waves) are known to emerge in non-linear media. The objective of WP4 is to demonstrate the microscopic counterpart of these phenomena, and to assess the possibility of tailoring the temporal entanglement of photon pairs by controlling their spatial degrees of freedom.

Year one was devoted to a theoretical description of the biphotonic correlation in the spatio-temporal domain, in various regimes and phase matching conditions of PDC. The research performed at COMO clearly outlined, for the first time to our knowledge, the non-separable geometry of the PDC entanglement with respect to spatial and temporal degrees of freedom. The name X-entanglement was coined to designate the X-shaped geometry of the biphoton state. The research demonstrated the relevant possibility of controlling the temporal bandwidth of biphotons by proper manipulation of their spatial degrees of freedom, thus achieving an unusual ultra-narrow temporal localization, in the femtosecond range, of photon pairs. [Gatti2009, Caspani2010, Brambilla2010a]. The work in year 2 addressed the crucial issue of identifying a measurement scheme able to demonstrate the X-entanglement. We concentrated on a setup where the biphoton spatio-temporal correlation is

measured by using the inverse process of sum frequency generation (SFG). We performed a careful modeling of the scheme [Brambilla2010b], which identifies the best conditions for the measurement and tackles several issues crucial for the forthcoming experimental implementation. Following these indications, a preliminary set up has been assembled at COMO [[Jedrkiwicz2010].

A quantum communication network is impossible without a quantum interface between light – the carrier of information – and matter – the storage medium for quantum information. Up to now, the work on light-atoms interfaces has been mostly limited to the case of a single spatial mode of light and a single spatial mode of atomic ensembles. Our **WP5**, *High-D entanglement of light and matter: quantum holograms* aims to extend this approach to multi-mode light-atom quantum interfaces. The main objective is to investigate multi-mode quantum memories for light based on spatially extended atomic ensembles, which we call quantum holograms, with the ultimate goal of an experimental demonstration of the extended spatial memory capacity of the quantum hologram, compared to a classical hologram.

A highlight theoretical result has been the theoretical development and critical comparison of various schemes for multi-mode quantum memories, including a thin quantum hologram with feedback [Vasilyev2009], a quantum volume hologram [Vasilyev2010], based on counter-propagating quantum signal wave and strong classical reference wave, and protocols based on the Faraday interaction or on *A*-type atomic level structures [Hammerer2010, Golubeva2009]. These studies, done in a joint effort of the USP, UPMC and UKBH partners, enabled us to understand how the various memory protocols work in an idealized limit, where the density of the storage medium is constant transverse to the propagation direction of light. In the second year we moved away from the ideal limit and the question of the memory efficiency in the presence of fluctuations and losses has been analyzed in 3-D models for different interaction schemes [Golubeva2010, Grodecka-Grad2010a]. The achieved results already deliver important inputs towards the evaluation of the information capacity of multimode memories and laid a firm foundation for the experimental work.

An experimental highlight result is achieved with the demonstration of a two-mode quantum memory, the work being now accepted for publication on Nature Physics [Jensen2010]. In a setup with room-temperature Cesium atoms, the write operation into a quantum memory for entangled two-mode states of light has been demonstrated. Storage of the quantum noise of two entangled light modes differing in frequency and interacting with two separate vapour cell is achieved in a configuration complementary to a spatial hologram – here different frequency components of single spatial mode light-field are stored in orthogonal spatial modes of atomic spin coherence.

The experimental study of decoherence and noise properties of quantum holographic media has been addressed also in setups with cold atoms. Spatially resolved Faraday imaging has been applied to assess the effective optical depth for an ultra-cold Rubidium sample [Kamiski2010]. In an independent experimental setup a cold Caesium sample has been employed to run an atomic clock protocol with spin-squeezed samples delivering valuable information about decoherence processes [Louchet-Chauvet2010].

## Publications, patents

- [Appel2009] J. Appel, P. J. Windpassinger, D. Oblak, U. B. Hoff, N. Kjærgaard, and E. S. Polzik, *Mesoscopic atomic entanglement for precision measurements beyond the standard quantum limit*, Proc. Nat. Ac. Sc. **106**, 10960 (2009).
- [Averchenko2010] V.A. Averchenko, Yu.M. Golubev, C. Fabre, N. Treps, “Quantum correlations and fluctuations in the pulsed light produced by a synchronously pumped optical parametric oscillator below its oscillation threshold”, Europ. Phys. J. D, in press (2010).
- [Beenakker2009] C.J.W. Beenakker, J.W.F. Venderbos and M.P. van Exter, Two-Photon Speckle as a Probe of Multi-Dimensional Entanglement, Phys. Rev. Lett. **102**, 193601 (2009).
- [Brambilla2010b] E. Brambilla, L. Caspani, L. Lugiato and A. Gatti, *Spatio-temporal structure of biphoton entanglement in type II PDC*, Phys. Rev. A **82**, 013835 (2010).
- [Brambilla2010] E. Brambilla, L. Caspani, O. Jedrkiewicz, J.-L. Blanchet, L. A. Lugiato, and A. Gatti. *Disclosing the spatio-temporal structure of PDC entanglement through frequency up-conversion*, preprint
- [Brida2009] G. Brida, L. Caspani, A. Gatti, M. Genovese, A. Meda, and I.R. Berchera, Measurement of Sub-Shot-Noise Spatial Correlations without Background Subtraction, Phys. Rev. Lett. **102**, 213602 (2009).
- [Caspani2010] L. Caspani, E. Brambilla and A. Gatti, *Tailoring the spatio-temporal structure of biphoton entanglement in type I PDC*, Phys. Rev A **81** 033808 (2010).
- [Chalopin2010a] B. Chalopin, A. Chiummo, C. Fabre, A. Maître and N. Treps, *Frequency doubling of low power images using a self-imaging cavity*, Opt. Exp. **18**, 8033 (2010).
- [Chalopin2010b] B. Chalopin, F. Scazza, C. Fabre and N. Treps, *A scalable source for multimode squeezing of transverse modes*, in preparation.

- [Chalopin2010c] B. Chalopin, F. Scazza, C. Fabre and N. Treps, *Multimode non-classical light generation through the OPO threshold*, Phys. Rev. A **81**, 061804(R) (2010).
- [Daems2010] D. Daems, F. Bernard, N. J. Cerf M. I. Kolobov, Tripartite entanglement in parametric down-conversion with spatially-structured pump, J. Opt. Soc. Am. B **27**, 447 (2010).
- [DiLorenzo2009a] H. Di Lorenzo Pires and M.P. van Exter, Observation of near-field correlations in spontaneous parametric down conversion, Phys. Rev. A **79**, 041801 (2009).
- [DiLorenzo2009b] H. Di Lorenzo Pires, C.H. Monkens and M.P. van Exter, Direct measurement of transverse-mode entanglement in two-photon states, Phys. Rev. A **80**, 022307(2009).
- [DiLorenzo2010] H. Di Lorenzo Pires, H.C.B. Florijn and M.P. van Exter, Measurement of the spiral spectrum of entangled two-photon states, Phys. Rev. Lett. **104**, 020505 (2010).
- [Gatti2009] A.Gatti, E.Brambilla, L. Caspani, O. Jedrkiewicz and L. Lugiato, X-Entanglement: the non factorable spatio-temporal structure of biphoton correlation, Phys. Rev. Lett. **102**, 223601 (2009).
- [Golubev2009] Yu. M. Golubev, T. Yu. Golubeva, A. A. Gavrikov and C. Fabre, Pure and mixed states in degenerate parametric oscillation, Opt. Spectr., **106** (5), 2009
- [Golubeva2009] T. Golubeva, Yu. Golubev, O. Mishina, J. Laurat, A Bramati, E. Giacobino, *Quantum hologram written by short pulse in resonant  $\Lambda$ -scheme*, in preparation.
- [Golubeva2010] T. Golubeva, Yu. Golubev, O. Mishina, A. Bramati, J. Laurat, E. Giacobino, *High speed spatially multimode atomic memory*, preprint
- [Grodecka-Grad2010a] A. Grodecka-Grad, E. Zeuthen, and A. S. Sørensen, *Three dimensional theory of spatial quantum memories based on  $\Lambda$ -type atomic ensembles*, preprint(2010).
- [Grodecka-Grad2010b] Anna Grodecka-Grad, Emil Zeuthen and Anders S. Sorensen, *Optimized interface with multimode quantum memories for entanglement storage*, in preparation.
- [Hammerer2010] K. Hammerer, A. S. Sørensen, and E.S. Polzik, *Quantum interface between light and atomic ensembles*, Rev. Mod. Phys. **82**, 1041–1093 (2010).
- [Kaminski2010] F. Kaminski, N. Kampel, A. Griesmaier, E. S. Polzik, and J. H. Müller, *Spatially resolved Faraday imaging of ultra-cold atomic samples*, report (2010) .
- [Jack2009] B. Jack, J. Leach, J. Romero, S. Franke-Arnold, M. Ritsch-Marte, S. M. Barnett, and M. J. Padgett, Holographic Ghost Imaging and the Violation of a Bell Inequality, Phys. Rev. Lett. **103**, 083602 (2009).
- [Janousek2009] J. Janousek, K. Wagner, J.F. Morizur, N. Treps, P.K. Lam, C.C. Harb, H.-A. Bachor, *Optical entanglement of co-propagating modes*, Nature Photonics **3**, 399 (2009).
- [Jedrkiewicz2010] O. Jedrkiewicz, J.-L. Blanchet, and P. Di Trapani, *Efficient pump reconstruction via ultra broadband sum frequency mixing of intense phase-conjugated twin beams*, submitted to Opt. Lett. (2010).
- [Jensen2010] K. Jensen, W. Wasilewski, H. Krauter, T. Fernholz, B. M. Nielsen, M. Owari, M. B. Plenio, A. Serafini, M. M. Wolf, and E. S. Polzik, *Quantum memory for entangled continuous-variable states*, Nature Physics, in press(2010)
- [Leach2009] J. Leach, B. Jack, J. Romero, M. Ritsch-Marte, R. W. Boyd, A. K. Jha, S. M. Barnett, S. Franke-Arnold, and M. J. Padgett, Violation of a Bell inequality in two-dimensional orbital angular momentum state-spaces, Opt. Express **17**, 8287-8293 (2009).
- [Leach2010] J. Leach, B. Jack, J. Romero, A. K. Jha, A. M. Yao, S. Franke-Arnold, D. Ireland, R. W. Boyd, S. M. Barnett, M. J. Padgett, Quantum Correlations in Optical Angle-Orbital Angular Momentum Variables, Science **329**, 662-66(2010).
- [Loeffler2010] W.Loeffler, T.G.Euser, E.R.Eliel, M.Scharrer, P.St.J.Russell and J.P.Woerdman, Transport of spatially entangled qutrits through a hollow-core photonic crystal fiber, arXiv:1007.4511
- [Lopez2009] L. Lopez, B. Chalopin, A. Rivière de la Souchère, C. Fabre, A. Maître and N. Treps, Multimode quantum properties of a self-imaging optical parametric oscillator: Squeezed vacuum and Einstein-Podolsky-Rosen-beams generation, Phys. Rev. A. **80**, 043816 (2009).
- [Leroyer 2010] Hadrien Leroyer, Claude Fabre, and Nicolas Treps Thomas Coudreau Jean-Pierre Gazeau, A symplectic toolbox for experimental quantum optics, in preparation.
- [Louchet-Chauvet2010] A. Louchet-Chauvet, J. Appel, J. Renema, D. Oblak, N. Kjærgaard, and E. S. Polzik, *Entanglement assisted atomic clock beyond the projection noise limit*, New J. of Phys. **12**, 065032 (2010) .
- [Morizur2010a] J.F. Morizur, L. Nicholls, P. Jian, S. Armstrong, N. Treps, B. Hage, M. Hsu, W. Bowen, J. Janousek and H.-A. Bachor, *Programmable unitary spatial mode manipulation*, JOSA A, in press
- [Morizur2010b] J.F. Morizur, S. Armstrong, N. Treps, J. Janousek and H.-A. Bachor, *Reshaping a continuous variable quantum state*, Europ. Phys. J. D, in press (highlight paper).
- [Oppo2009] G.-L. Oppo, D. Cuozzo, G. D’Alessandro, F. Papoff, and J. Jeffers, Macroscopic Continuous Variable Entanglement of Giant Sub-threshold Pulses in SPOPO, in preparation.
- [Patera2009] G. Patera, N. Treps, C. Fabre, Tailoring the quantum properties of supermodes for quantum information applications, in preparation
- [Patera2010a] G. Patera, G. De Valcarcel, N. Treps, C. Fabre, Quantum theory of synchronously pumped type I Optical Parametric Oscillators: characterization of the squeezed supermodes, Europ. Phys. J. D **56**, 123 (2010).

- [*Patera2010b*] G. Patera, M. I. Kolobov, Spatio-temporal properties of multipartite entanglement, Proc. SPIE Vol. 7727, 77270I (2010).
- [*Peeters2009*] W.H. Peeters, J.J. Renema and M.P. van Exter, Engineering two-photon quantum correlations in spontaneous parametric down conversion, Phys. Rev. A **79**, 043817(2009)
- [*Pinel2009*] O. Pinel, B. Chalopin, N. Treps, French patent pending n° 09 04537, submitted on 23/09/09: «Procédé de stabilisation de la longueur d'une cavité optique»
- [*Pinel2010*] O. Pinel, J. Fade, N. Treps and C. Fabre, General Cramer-Rao bound for parameter estimation using Gaussian multimode quantum resources, submitted to Phys. Rev. Letters (2010).
- [*Pors2009*] B.J.Pors, C.H.Monken, E.R.Eliel and J.P.Woerdman, Transport of OAM entanglement through a turbulent atmosphere, e-print [arXiv:0909.3750v1](https://arxiv.org/abs/0909.3750v1)
- [*Sørensen2009*] Martin W. Sørensen and Anders S. Sørensen, Three-dimensional theory of stimulated Raman scattering. Phys. Rev. A **80**, 033804 (2009).
- [*Vasilyev2009*] D.V.Vasilyev, I.V.Sokolov, and E.S.Polzik, Quantum Memory for Images with Feedback, Optics and Spectroscopy **106**, 875 (2009).
- [*Vasilyev 2010*] D. V. Vasilyev, I. V. Sokolov, and E. S. Polzik, *Quantum volume hologram*, Phys. Rev. A **81**, 020302(R) (2010); D. V. Vasilyev, I. V. Sokolov, and E. S. Polzik, *Double-pass quantum volume hologram*, preprint (2010).